Tentamen Foutenanalyse 7-2-2012 uitwerking

1a) $L = (1{,}000 \pm 0{,}001)\ \mathrm{m}$

b) $C = (1{,}4 \pm 0{,}5)\ \mathrm{mF}$

c) $L = (0{,}96 \pm 0{,}04)\ \mathrm{m}$

d) $R = (85000 \pm 30)\mathrm{k\Omega}$ *of* $R=(8{,}500 \pm 0{,}003).10^6\ \Omega$ *of* $R=(8{,}500 \pm 0{,}003)\ \mathrm{M\Omega}$

2. $$\frac{\Delta P}{P} = \left[4\left(\frac{\Delta I}{I}\right)^2 + \left(\frac{\Delta R}{R}\right)^2\right]^{\frac{1}{2}} = \left[4\left(\frac{0{,}01}{0{,}34}\right)^2 + 0{,}05^2\right]^{\frac{1}{2}} = 0{,}058 \quad \to \quad \Delta P = 0{,}058\,P$$

$$P = I^2R = 0{,}34^2 \cdot 47 = 5{,}4332\ W$$

$$\Delta P = 0{,}058\,P = 0{,}058 \cdot 5{,}4332 = 0{,}315 \approx 0{,}4\ W$$

$$P = (5{,}4 \pm 0{,}4)\ W$$

3a) $$\int_{-\infty}^{\infty} f(x)dx = 1 \ \to\ A\int_0^2 (4x^2 - x^4)dx = 1 \ \to A\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = A\left\{\frac{32}{3} - \frac{32}{5}\right\} = 1 \ \to$$

$$A\frac{64}{15} = 1 \ \to A = \frac{15}{64}$$

b) $$\bar{x} = \int_{-\infty}^{\infty} xf(x)dx = \frac{15}{64}\int_0^2 (4x^3 - x^5)\,dx = \frac{15}{64}\left[x^4 - \frac{1}{6}x^6\right]_0^2 = \frac{15}{64}\left\{16 - \frac{64}{6}\right\} = \frac{5}{4}$$

c) $$\overline{x^2} = \int_{-\infty}^{\infty} x^2 f(x)dx = \frac{15}{64}\int_0^2 (4x^4 - x^6)\,dx = \frac{15}{64}\left[\frac{4}{5}x^5 - \frac{1}{7}x^7\right]_0^2 = \frac{15}{64}\left\{\frac{128}{5} - \frac{128}{7}\right\} = \frac{12}{7}$$

$$\sigma = \sqrt{\overline{x^2} - \bar{x}^2} = \sqrt{\frac{12}{7} - \left(\frac{5}{4}\right)^2} = \sqrt{\frac{17}{112}} \approx 0{,}39$$

4a) $$\overline{g_1} = \frac{9{,}83+9{,}78+9{,}80+9{,}86+9{,}79}{5} = 9{,}812\ ms^{-2}$$

b) $$s_g = \sqrt{\frac{\sum(g_i - \bar{g})^2}{N(N-1)}} = \sqrt{\frac{0{,}018^2+0{,}032^2+0{,}012^2+0{,}048^2+0.022^2}{5 \cdot 4}} = 0{,}0146 \approx 0{,}02\ ms^{-2}$$

$$\to g = (9{,}81 \pm 0{,}02)\ ms^{-2}$$

c) $g_1 = (9{,}81 \pm 0{,}02)\ ms^{-2}$ $g_2 = (9{,}78 \pm 0{,}06)\ ms^{-2}$

$$w_1 = \frac{1}{(0{,}02)^2} = 2500\ ; \quad w_2 = \frac{1}{(0{,}06)^2} = 278\ ;$$

$$\bar{g} = \frac{w_1 g_1 + w_2 g_2}{w_1 + w_2} = \frac{2500 \cdot 9{,}81 + 278 \cdot 9{,}78}{2500 + 278} = 9{,}807\ ms^{-2}$$

d) $s_{\bar{g}} = \sqrt{\frac{1}{w_1 + w_2}} = \sqrt{\frac{1}{2778}} = \sqrt{0{,}00036} = 0{,}01897 \approx 0{,}02\ ms^{-2}$

$\rightarrow \quad g = (9{,}81 \pm 0{,}02) ms^{-2}$

5a) $a = \frac{\overline{xy} - \bar{x}\,\bar{y}}{\overline{x^2} \; - \bar{x}^2}$ $\qquad b = \bar{y} - \; a\bar{x}$

x_i	y_i	$x_i y_i$	x_i^2
1	1	1	1
2	2	4	4
3	4	12	9
4	5	20	16
5	6	30	25
$\bar{x} = 3$	$\bar{y} = 3{,}6$	$\overline{xy} = 13{,}4$	$\overline{x^2} = 11$

$\rightarrow \quad a = \frac{13{,}4 - 3 \cdot 3{,}6}{11 - 9} = \frac{2{,}6}{2} = 1{,}3$ $\qquad b = 3{,}6 - 1{,}3 \cdot 3 = -\,0{,}3$

b)

x_i	y_i	$D_i = y_i - a x_i - b$	D_i^2
1	1	0	0
2	2	-0,3	0,09
3	4	0,2	0,04
4	5	0,1	0,01
5	6	-0,2	0,04
			$\sum D_i^2 = 0{,}18$

$\sigma^2 = \frac{\sum D_i^2}{N - 2} = \frac{0{,}18}{3} = 0{,}06 \quad \rightarrow \quad \sigma = 0{,}24$

$\sigma_a^2 = \frac{\sigma^2}{N\left(\overline{x^2} - \bar{x}^2\right)} = \frac{0{,}06}{5(11 - 9)} = 0{,}006 \;\; \rightarrow \;\; \sigma_a \; = 0{,}077 \approx 0{,}1$

$\sigma_b^2 = \frac{\overline{x^2}\sigma^2}{N\left(\overline{x^2} - \bar{x}^2\right)} = \frac{11 \cdot 0{,}06}{5(11 - 9)} = 0{,}066 \;\; \rightarrow \;\; \sigma_b \; = 0{,}257 \approx 0{,}3$

$\rightarrow \quad a = 1{,}3 \pm 0{,}1 \quad en \quad b = \; -0{,}3 \pm 0{,}3$